GRAIL gravity field determination using the Celestial Mechanics Approach - status report

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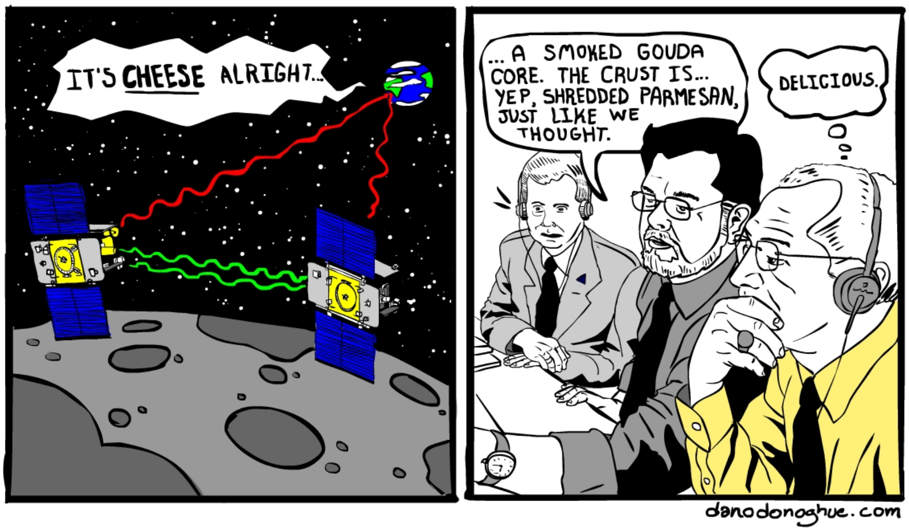
Outline

The GRAIL mission

Science objectives

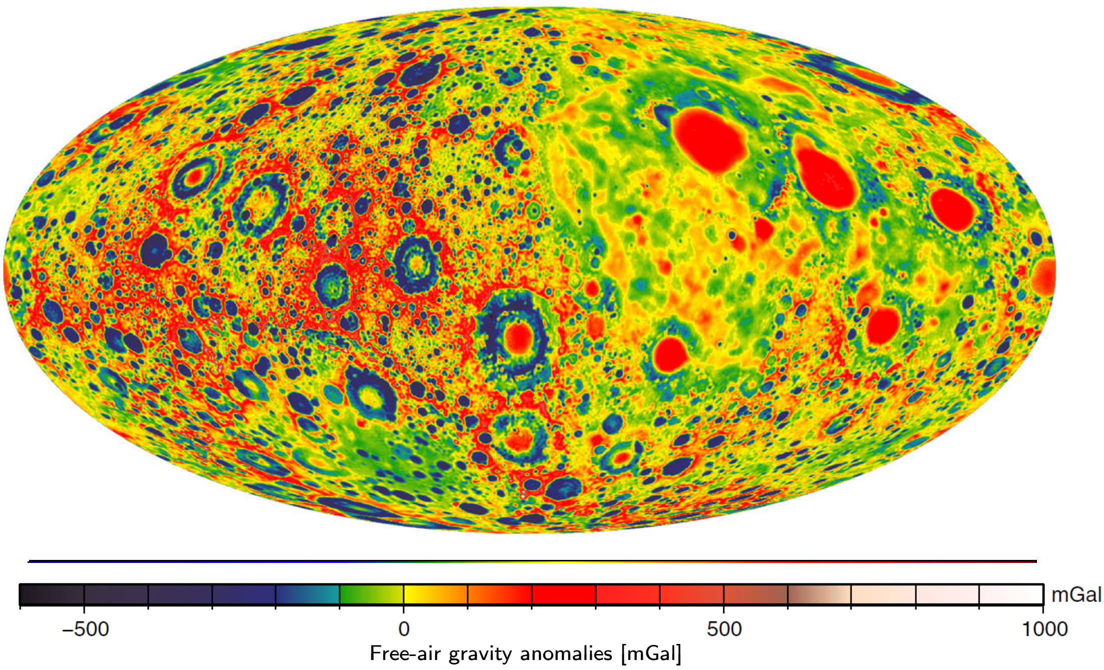


- Determine structure of lunar interior, from crust to core
  - → Subsurface structure of impact basins, mascons, ...
- Understand (asymmetric) thermal evolution of Moon

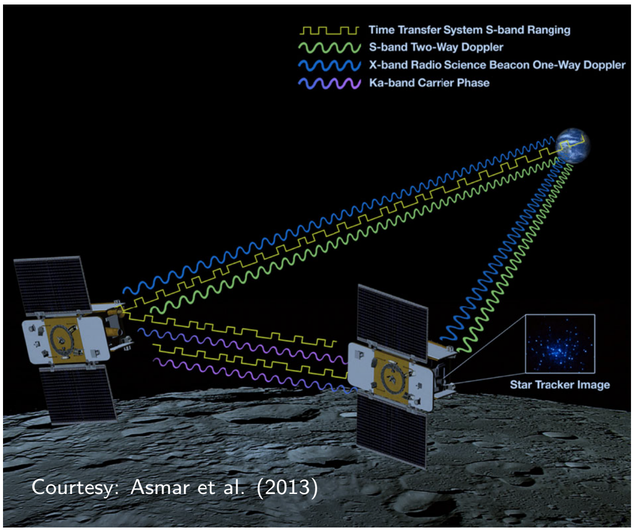
The GRAIL mission

GRAIL: latest official $l_{max} = 900$ gravity field models:
GRGM900C (Lemoine et al., 2014), GL0900C (Konopliv et al., 2014)

Pre-GRAIL lunar gravity missions: Lunar Prospector (NASA, 1998-99)
SGL165P1



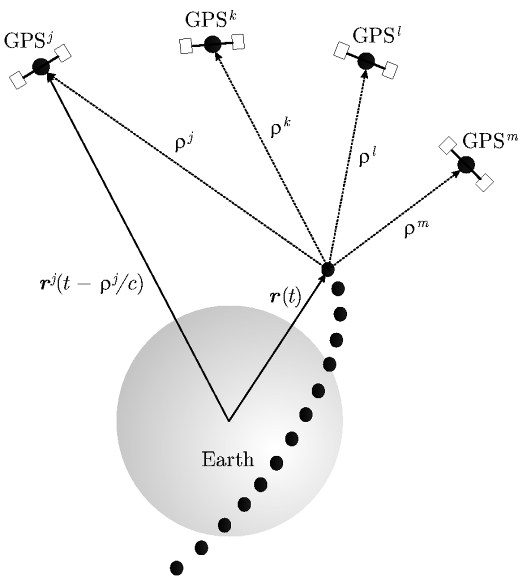
The GRAIL mission: Satellite signals



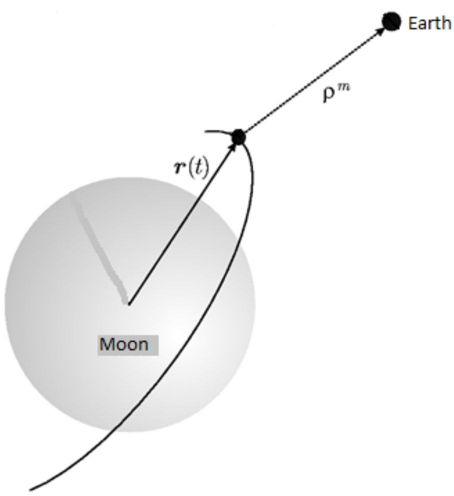
- S-band ($\sim 2\,\mathrm{GHz}$) for 2-way Doppler tracking by NASA Deep Space Network (DSN)
- X-band ($\sim 8\,\mathrm{GHz}$) for 1-way Doppler tracking
- Ka-band ($\sim 32\,\mathrm{GHz}$) inter-satellite link

Our motivation: Why not adapt our procedures for the processing of GRACE data (K-band etc.) to GRAIL, get experienced in this new environment and eventually provide an independent lunar gravity field solution?

GRACE vs. GRAIL



GRACE: Kinematic positions using GPS observations



GRAIL: DSN Doppler tracking (near-side only) yields positions

The GRAIL mission: Available data

Selection of available data for our activities:

- 1- and 2-way Doppler data
- Ka-band range data: Ka-band range rate (KBRR)
  - → 5 s-sampling in primary, 2 s-sampling in extended mission phase
- Reduced-dynamic positions (GNI1B) of GRAIL-A and GRAIL-B (by-product of gravity field estimation)
  - → 5 s-sampling in primary and extended mission phase

Using the GNI1B positions as pseudo-observations allows us to gain first experience in GRAIL orbit and gravity field determination without the necessity to process DSN data!

However: Not independent! → new Doppler capability in BSW

The Celestial Mechanics Approach

(implemented in the Bernese GNSS Software)

The Celestial Mechanics Approach (CMA)

Selenocentric equation of motion for satellite i

$$\ddot{\mathbf{r}}_i = -GM_M \frac{\mathbf{r}}{r^3} + \mathbf{f}(t, \mathbf{r}, \dot{\mathbf{r}}, q_1, ..., q_d)$$

$$\mathbf{f} = \nabla V + \mathbf{a}_b + \mathbf{a}_t + \mathbf{a}_r + \mathbf{a}_e + \mathbf{a}_n$$

V Lunar gravity potential:

$$V(r,\lambda,\phi) = \frac{GM_M}{r} \sum_{l=1}^{l_{\max}} \left(\frac{R_M}{r}\right)^l \sum_{m=0}^{l} \bar{P}_{lm}(\sin\phi) \left(\bar{C}_{lm}\cos m\lambda + \bar{S}_{lm}\sin m\lambda\right)$$

$\mathbf{a}_b$ 3rd body perturbations (Earth, Sun, Jupiter, Venus, Mars, according to JPL ephemerides DE421)

$\mathbf{a}_t$ Tidal deformation of Moon due to Earth and Sun. IERS2010 conventions:

$$\Delta\bar{C}_{lm} - i\Delta\bar{S}_{lm} = \frac{k_{lm}}{2l+1} \sum_{j=2}^{3} \frac{GM_j}{GM_M} \left(\frac{R_M}{r_j}\right)^{l+1} \bar{P}_{lm}(\sin\Phi_j) e^{-im\lambda_j}$$

Stefano Bertone: GRAIL gravity field determination using the Celestial Mechanics Approach - status report
EPSC 2015, 30 September, Nantes (France)

The Celestial Mechanics Approach (CMA)

- Development version of Bernese GNSS software.
- Linearization of orbit around a priori orbit.
- Numerical integration (with a priori parameters) of equations of motion and variational equations.
- Set up of position and Ka-band normal equations (NEQs) on a daily basis.
- Combination of position and Ka-band NEQs with appropriate weighting.
- NEQ manipulation: Preelimination of parameters and accumulation to weekly, monthly and three-monthly NEQs, which are then inverted without applying any regularization.

All parameters estimated simultaneously

$\longrightarrow$ Gravity field estimation = extended orbit determination problem

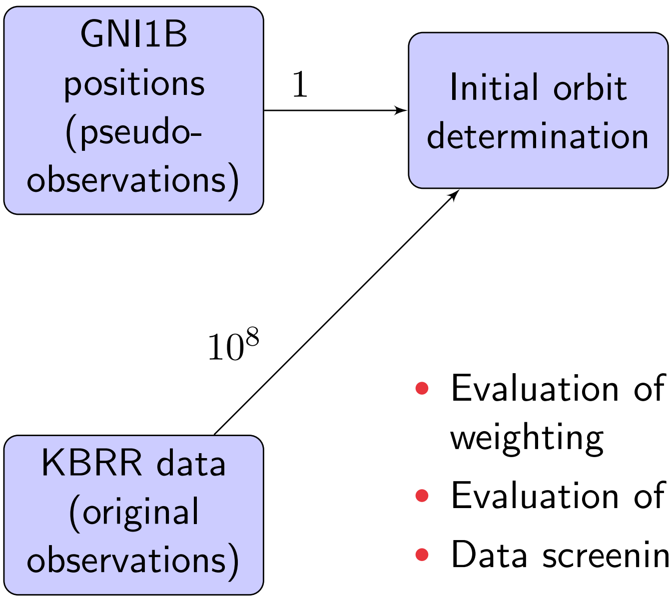
Results using GNI1B+KBRR data

(Based on release-4 data of primary mission phase)

[Arnold, D., Bertone, S., Jäggi, A., Beutler, G. and Mervart, L.
GRAIL gravity field determination using the Celestial Mechanics Approach,
Icarus, 2015]

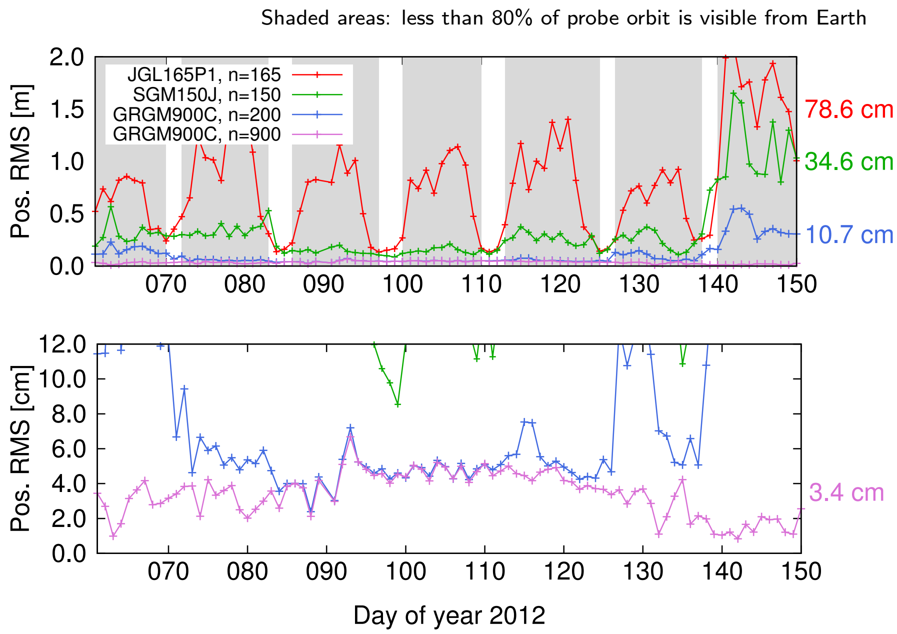
Orbit determination

Use the GNI1B positions as pseudo-observations for an initial orbit determination for GRAIL-A and GRAIL-B.
Add the Ka-band range rate data to improve orbit determination.



- Evaluation of the appropriate relative weighting
- Evaluation of optimal pulses spacing
- Data screening based on KBRR residuals

Orbit determination: Positions only



Daily RMS values of GNI1B position fit over the whole primary mission phase, using different gravity field models. Slightly worse fits for beginning and end of primary mission phase when using GRGM900C to $n = 200$.

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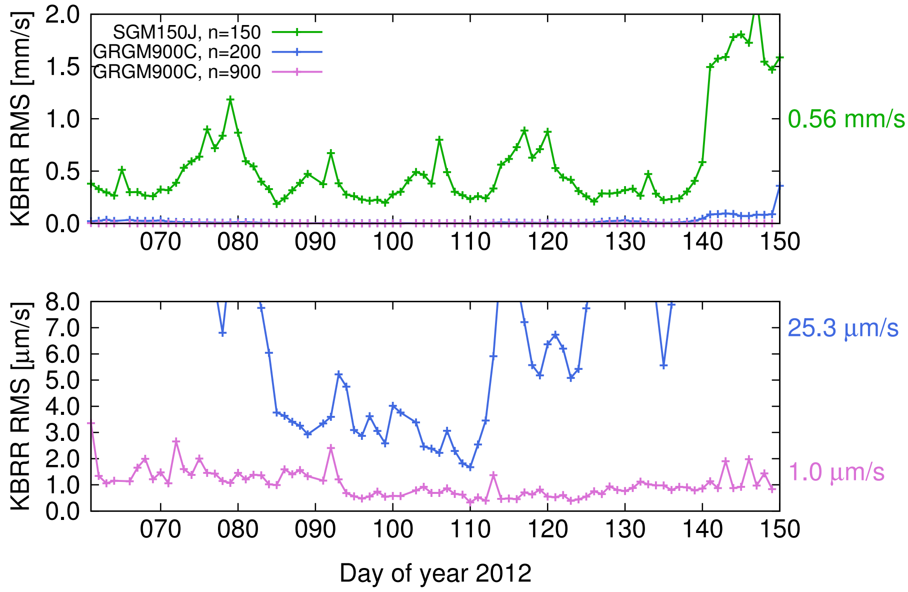
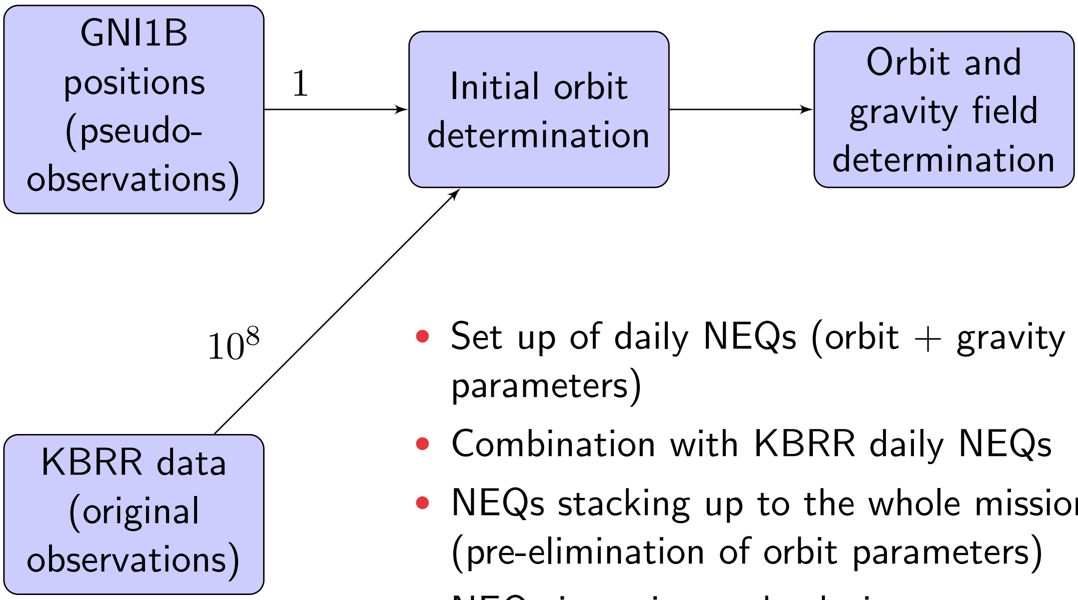
Orbit determination: Combined



Daily RMS values of KBRR residuals over the whole primary mission phase, using different gravity field models.

Gravity field determination

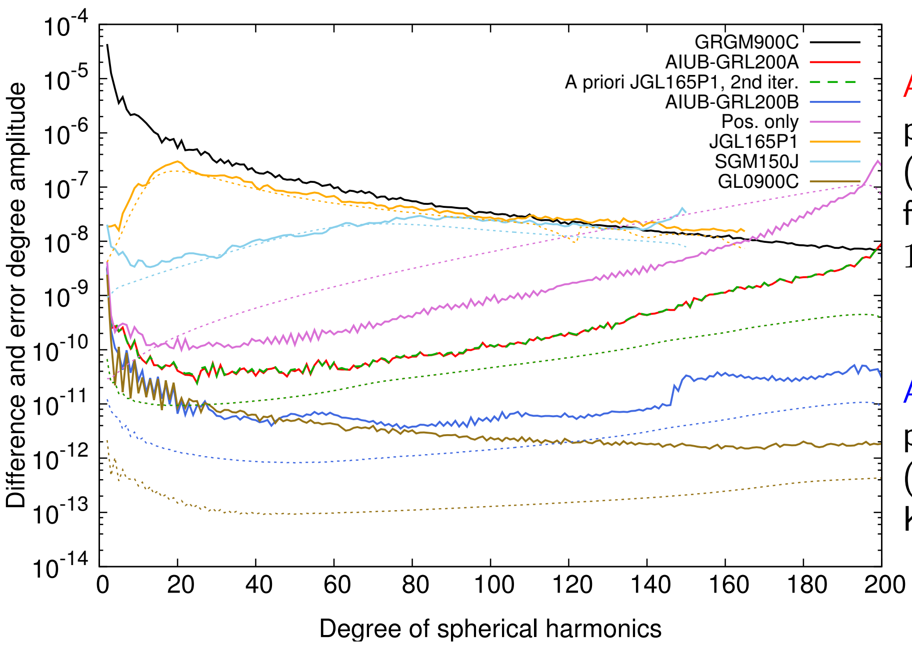
Use initial orbits for a combined orbit and gravity field determination



- Set up of daily NEQs (orbit + gravity field parameters)
- Combination with KBRR daily NEQs
- NEQs stacking up to the whole mission (pre-elimination of orbit parameters)
- NEQs inversion and solution

Gravity field determination: Up to $l_{\max} = 200$

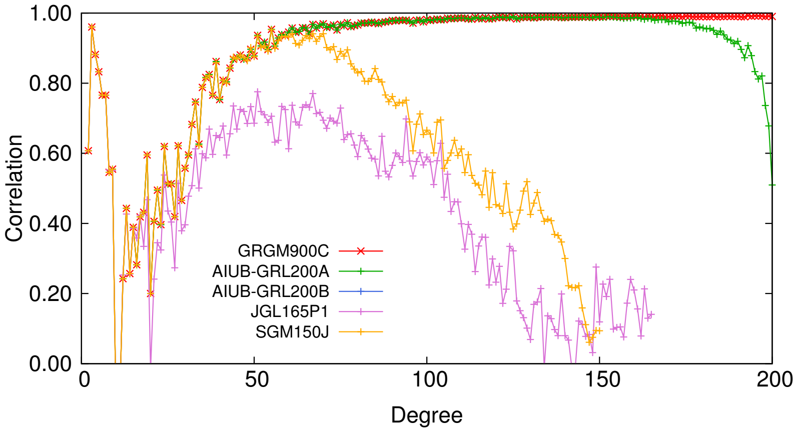
Difference degree amplitudes : $\Delta_l = \sqrt{\frac{1}{2l+1}\sum_{m=0}^{l}\left(\Delta\bar{C}_{lm}^2 + \Delta\bar{S}_{lm}^2\right)}$



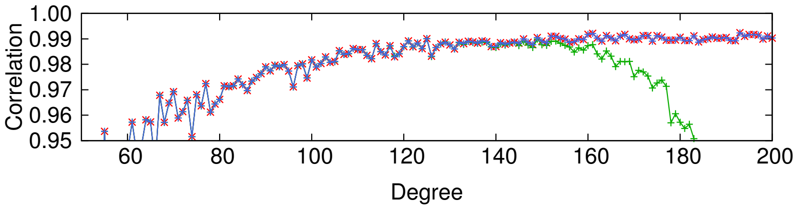
AIUB-GRL200A : a priori GRGM900C (d/o 200), post-fit KBRR = $19.7\ \mu m/s$

AIUB-GRL200B : a priori GRGM900C (d/o 600), post-fit KBRR = $0.7\ \mu m/s$

Correlation between gravity and topography



AIUB-GRL200A : $> 98\%$ up to d/o 150



AIUB-GRL200B : $> 98\%$ up to d/o 200

"Validation" against gravity field induced by LOLA (LRO) topography.

 Astronomical Institute, University of Bern AIUB

Results using Doppler+KBRR data
(New implementation in Bernese GNSS Software, preliminary study)

Gravity field determination

Goal: Replace GNI1B positions by original DSN Doppler observations.

Doppler data (original observations)

Initial orbit determination

Orbit and gravity field determination

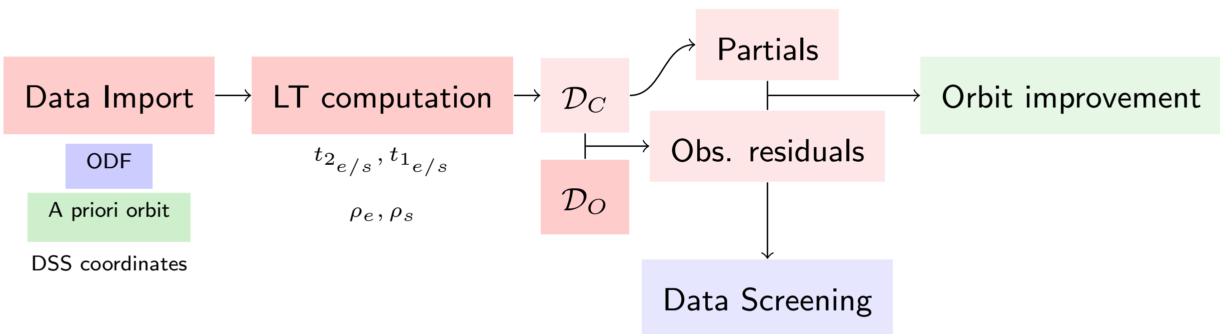
KBRR data (original observations)

- **A priori orbit: fit of Doppler observations with appropriate parametrization**
- Combination with KBRR daily NEQs
- NEQs stacking up to the whole mission (pre-elimination of orbit parameters)
- NEQs inversion and solution

DSN Doppler data processing

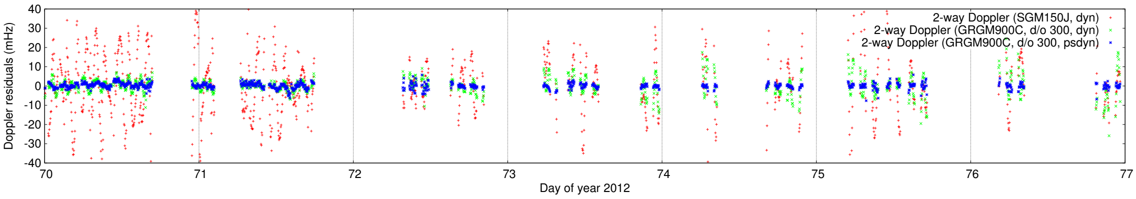
Doppler model based on [Moyer, 2000]. It includes :

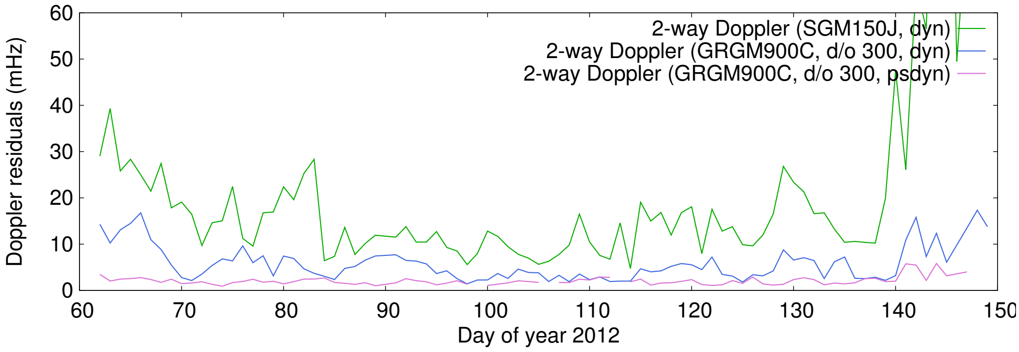
- Tracking stations Earth-fixed coordinates
- Earth rotation (IERS2010)
- planetary ephemeris (DE421, . . .)
- IAU2010 (time scales, . . .)
- Relativistic effects (Shapiro, . . .)
- Atmospheric delay (troposphere, ionosphere)



Two-way Doppler residuals

Doppler observations are not continuous nor regularly spaced.





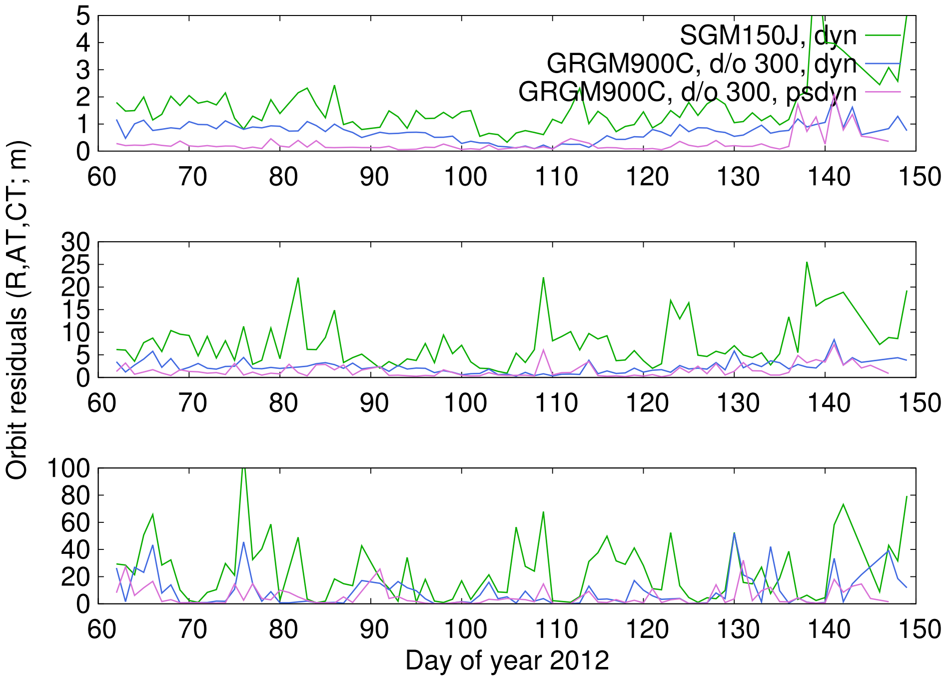
34 mHz

7 mHz

2 mHz

Daily RMS values of Doppler residuals of GRAIL-A (primary mission, PM), using SGM150J and GRGM900C up to d/o 300 and different parametrizations.

Orbit residuals: doppler only



Radial
1.9 m

0.78 m
0.4 m

Along-track
9.2 m

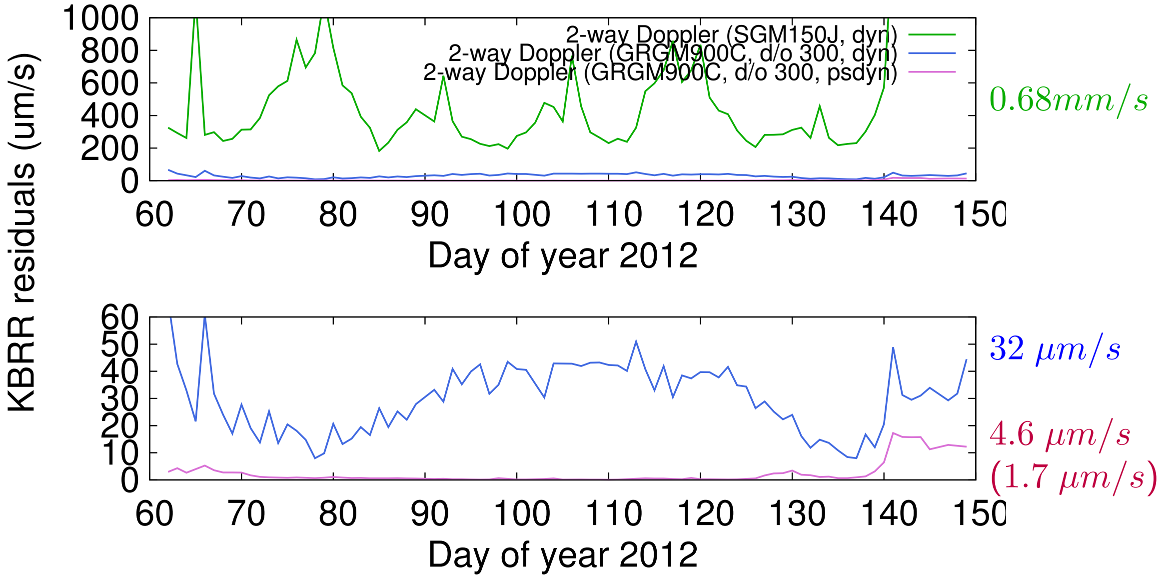
2.7 m
2.1 m

Cross-track
31.9 m

14 m
8.3 m

Daily RMS values of orbital fit of GRAIL-A (PM), using SGM150J and GRGM900C up to d/o 300 with different parametrizations.

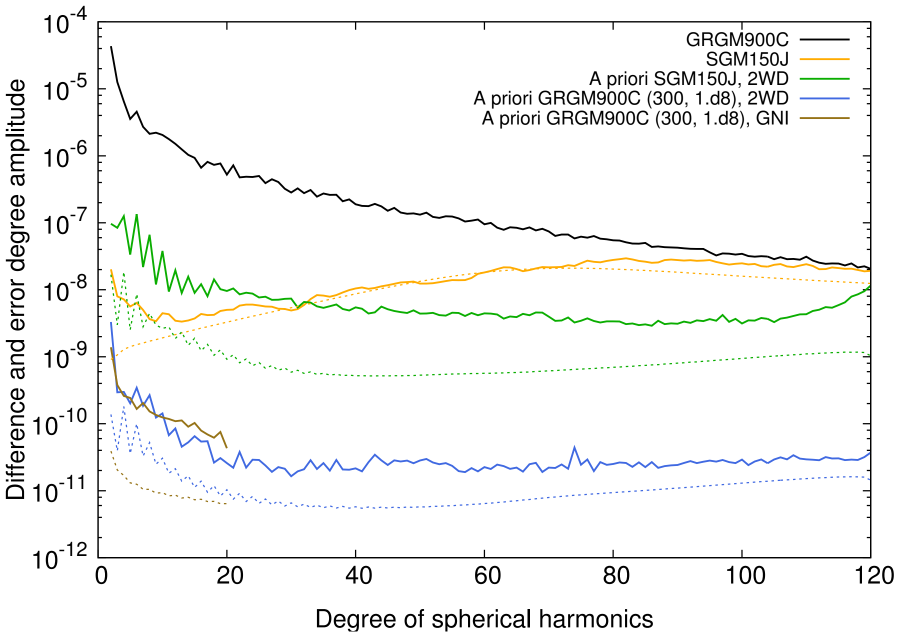
Combined orbit: KBRR residuals



Daily RMS values of KBRR residuals for GRAIL-A (PM), using different gravity field models and parametrizations.

- Days $140 - 150$ at lower altitude $\rightarrow$ larger residuals

Gravity field determination



a priori SGM150J: bad low degrees, more iterations needed

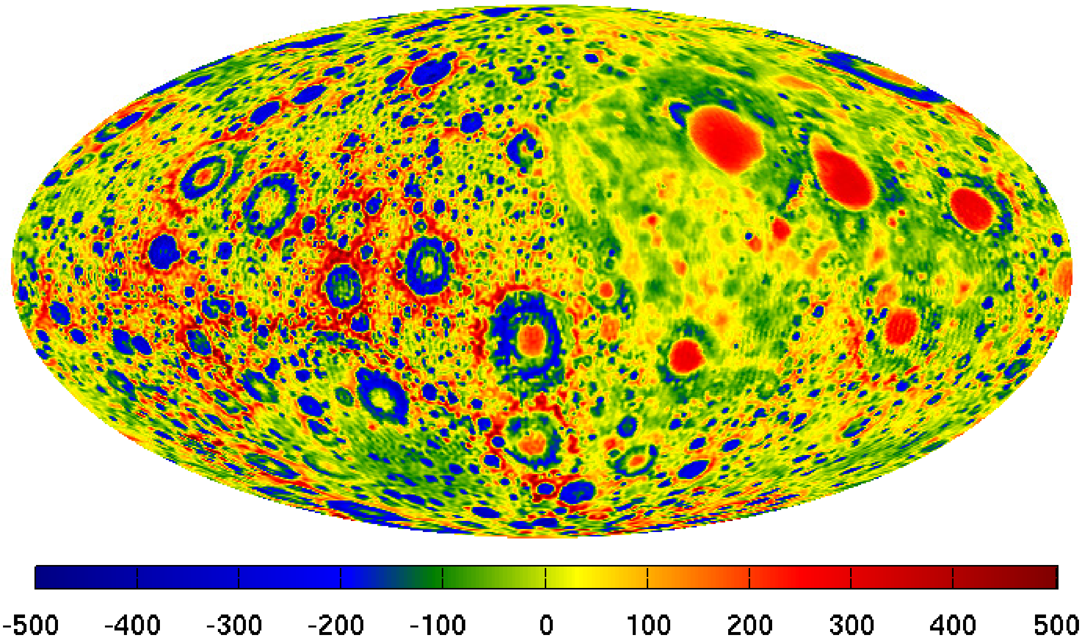
a priori GRGM900C (d/o 300): same level as GNI+KBRR solution

- First d/o 120 solution from original observations
- Further investigations are needed to improve solutions starting from "poor" a priori gravity fields.

Conclusion & Outlook

Conclusion & Outlook

- Due to availability of GNI1B positions the adaption of the CMA from GRACE to GRAIL is feasible without DSN data analysis.
- Pseudo-stochastic orbit parametrization allows for "Bernese" lunar gravity fields without sophisticated background models.
- We reach the μm/s level for KBRR residuals. But radiation pressure modelling is crucial to further improve the solutions.
- New Doppler orbit determination capabilities in Bernese GNSS Software (future application to other planetary missions).
- Orbit parametrization and arc-length need to be optimised for new scenario.
- First fully independent "Bernese" lunar gravity field solutions.



AIUB-GRL200A: Free-air gravity anomalies [mGal]

Thank you!